Cognitive Abilities Test™
Practice Activities
Teacher Guide

Form 7

CogAT®

Test 4: Number Analogies, Levels 10–12

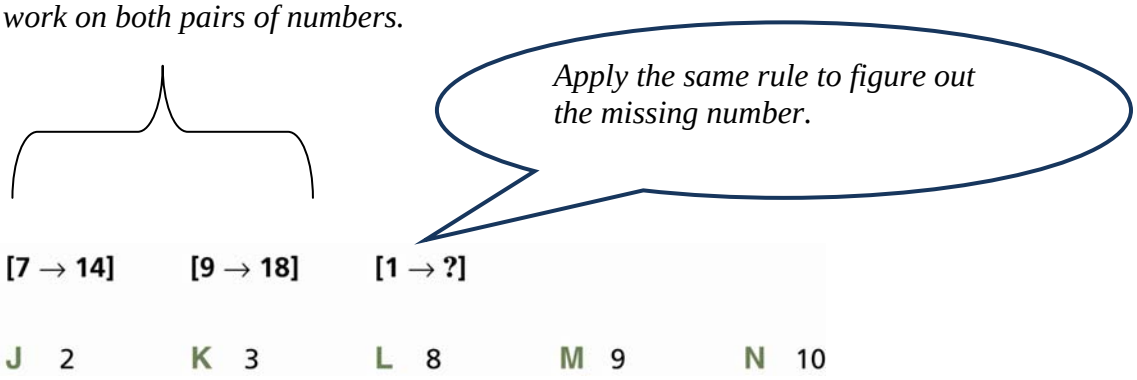
Part 1: Overview of Number Analogies

An analogy draws parallels between objects or ideas. Successful learners habitually reason by analogy. Good analogies allow students to use what they already know when they are trying to understand or remember new ideas. Reasoning by analogy requires attending carefully to the ways in which two things are similar. Then this relationship is mapped onto something new. Performance on this test predicts mathematics achievement because discovering quantitative patterns and relationships is at the core of learning mathematics.

Each question on the Number Analogies test presents two pairs of numbers. To solve the questions, students must find a rule that transforms the first number in each pair into the second number. The same rule must work for both pairs of numbers. Then the student uses the rule to generate the missing number in the third pair. Most questions require only one rule, but some questions use two rules such as adding and then doubling. Students are allowed to use scratch paper on the actual test. Items in the test look like this:

What rule explains how to transform 7 into 14 and 9 into 18? The rule must work on both pairs of numbers.

Apply the same rule to figure out the missing number.



When practicing the Number Analogies questions, encourage students to use these strategies.

- Think of a rule that describes how to transform the first number into the second number in the first two pairs. For example, the second number is double the first.
- If more than one rule works, then try the simplest rule first.
- Apply the same rule to the third pair of numbers to determine the missing number.

Students at this level tend to make the following common mistakes.

- Students may use the wrong arithmetic operation. For example, the student might add instead of subtract.

- Students might find a rule that works with only one of the pairs and not both. For example, in the sample question above, a student may remember that adding 7 to 7 will make 14. Although this rule does not work for the second pair, the student may simply add 7 to 1, and marks the answer as 8.

Part 2: Number Analogies Practice Test Script

The following script covers many issues that will help students do their best on the test. Read aloud the text printed in *blue italics*: these are directions to the students. Directions for you are in parentheses and should NOT be read aloud. Feel free to modify the script to ensure that students understand what they are supposed to do and how do it.

It may be helpful to make copies of the practice questions in order to display them one at a time on an overhead projector. If this is not possible, hold up a copy of the student practice booklet and point to different parts of each practice question as you discuss them with the class.

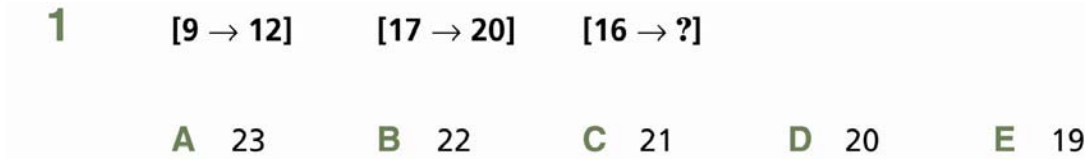
(Make sure each student has a practice booklet. Then **SAY**:)

Open your practice booklet to page 1.

(Check that all students have the correct page.)

P1

Let's do the first practice question. Look at the numbers in the first row.



(Hold up your copy of the booklet. Point to the first row of the question as you **SAY**:)

Each of the pairs of numbers in the first row has an arrow between the numbers. The arrows mean that the two numbers must follow a rule.

How do we go from 9 to 12 and from 17 to 20?

(Encourage responses.)

In this question, the rule for each pair is to add 3 to the first number to get the second number. In the first pair, 9 plus 3 is 12. In the second pair, 17 plus 3 is 20.

Look at the third pair of numbers in the top row. This pair must follow the same rule. What is the missing number?

(Encourage responses.)

The rule is to add 3 to the first number. 16 plus 3 is 19. So the missing number is 19 which is answer choice E.

(Check to make sure that all students have selected answer choice E.)

P2

Now let's do the second practice question.

What rule do the first two pairs of numbers follow?

(Encourage responses. If a student suggests "add 7 to the first number" as the rule, **SAY**:)

The rule "add 7 to the first number" works for the first pair of numbers. But does it work for the next pair?

(Encourage responses.)

No, it doesn't. If we add 7 to 9 we get 16, not 18. So this rule does not work.

Is there another rule we can use?

(Encourage responses.)

In both pairs, when we double the first number we get the second number. When we double 7 we get 14 and when we double 9 we get 18.

Using this rule, what is the missing number in the third pair?

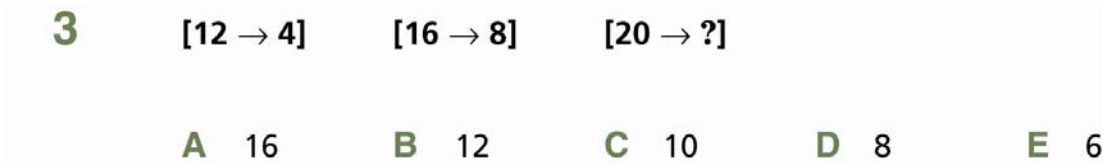
(Encourage responses.)

When we double 1 we get 2. So the missing number is 2 which is answer choice J.

(Check to make sure that all students have selected answer choice J.)

P3

Now look at the next practice question. Try to solve this practice question on your own.



(Make sure students have enough time to solve the problem. Then **SAY**:)

What is the missing number in the third pair?

(Encourage responses.)

How did you solve this question?

(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

What rule do the first two pairs of numbers follow?

(Encourage responses.)

In this question, the rule for each pair is to subtract 8 from the first number to get the second number. When we subtract 8 from 12 we get 4 and when we subtract 8 from 16 we get 8.

We must follow the same rule for the third pair. What is the missing number?

(Encourage responses.)

The rule is to subtract 8 from the first number. 20 minus 8 is 12. So the missing number is 12 which is answer choice B.

(Check to make sure that all students have selected answer choice B.)

P4

Now look at the next practice question. Try to solve this practice question on your own.

(Make sure students have enough time to solve the problem. Then **SAY**:)

What is the missing number in the third pair?

(Encourage responses.)

How did you solve this question?

(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

What rule do the first two pairs of numbers follow?

(Encourage responses.)

In both pairs, when we divide the first number by 2 (or halve it) we get the second number.

(Note that the rule could also be "the second number is half of the first number.")

In the first pair, when we divide 18 by 2 we get 9. In the second pair, when we divide 12 by 2 we get 6. Using this rule, what is the missing number in the third pair?

(Encourage responses.)

When we divide 24 by 2 we get 12. So the missing number is 12 which is answer choice N.

(Check to make sure that all students have selected answer choice N.)

P5

Now let's try the next question. This one is more difficult.

How do we go from 10 to 22 and from 8 to 18?

(Encourage responses.)

Let me give you a hint: The rule for each pair in this question requires two changes.

(Encourage students to discover the two rules.)

The rule for this question is to multiply the first number by 2 and then add 2 to get the second number. In the first pair, we multiply 10 by 2 to get 20 and then 20 plus 2 is 22.

Let's see if this rule works for the second pair as well.

In the second pair, we multiply 8 by 2 to get 16 and then 16 plus 2 is 18.

Now look at the third pair. We must follow the same rule. What is the missing number?

(Encourage responses.)

When we multiply 4 by 2 we get 8 and then 8 plus 2 is 10. So the missing number is 10 which is answer choice D.

(Check to make sure that all students have selected answer choice D.)

P6

Look at the last practice question. Try to solve this practice question on your own.

(Make sure students have enough time to solve the problem. Then **SAY**:)

What is the missing number in the third pair?

(Encourage responses.)

How did you solve this question?

(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

The rule for each number pair in this question requires two changes. What is the rule?

(Encourage responses.)

The rule for this question is to multiply the first number by 3 and then subtract 1 to get the second number. In the first pair, we multiply 7 by 3 to get 21 and then 21 minus 1 is 20.

A rule that works for the first pair also must work for the second pair.

In the second pair, we multiply 1 by 3 to get 3 and then 3 minus 1 is 2.

We must follow the same rule with 4 in the third pair. What is the missing number?

(Encourage responses.)

When we multiply 4 by 3 we get 12 and then 12 minus 1 is 11. So the missing number is 11 which is answer choice M.

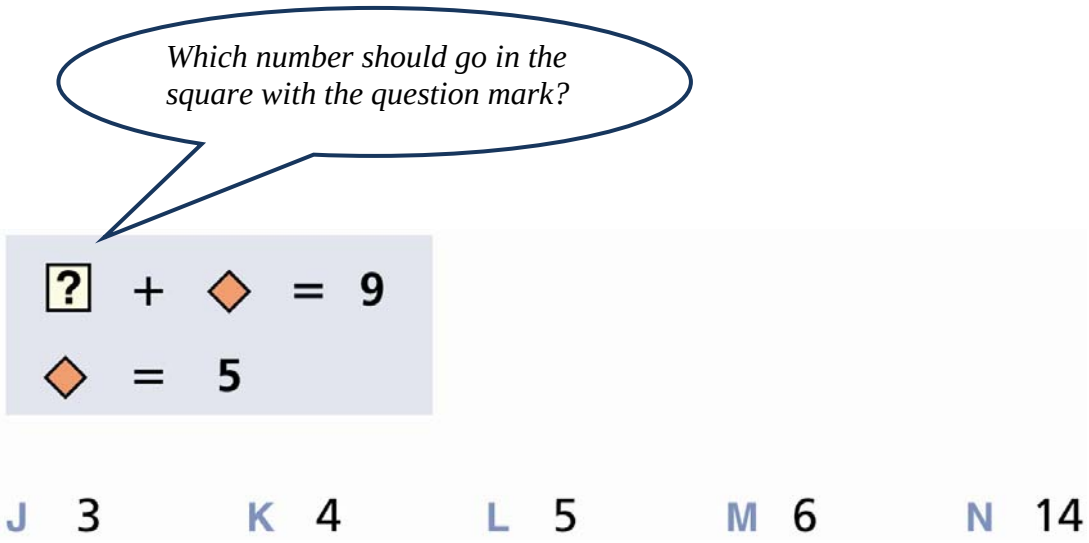
(Check to make sure that all students have selected answer choice M.)

Test 5: Number Puzzles, Levels 10–12

Part 1: Overview of Number Puzzles

The Number Puzzles test requires the student to solve simple equations by choosing an answer that makes the numbers on both sides of the equal sign the same amount. To answer the questions in this test, the student must find the missing number that goes in the square with the question mark. This test measures understanding of mathematical identities.

In this test, students are asked to solve problems that look like this:



When practicing the Number Puzzles questions, encourage students to use these strategies.

- Perform the necessary calculations to make both sides of the equal sign total the same amount in each row of the question.
- Substitute numbers for shapes wherever possible. For example, in the above sample question, the second row shows that the diamond is equal to 5. The number 5 can replace the diamond in the top row.
- Use scratch paper to keep track of each step in the solution.

Students at this level tend to make the following common mistakes.

- Students may work from the top down rather than from the bottom up. Solving the simple identities in the second and third lines is necessary before one can solve the identity in the first line.

- Students might use the wrong arithmetic operation. For example, the student may add numbers instead of subtracting them.

- Students may solve the question for a shape other than the square with the question mark. For example, in the sample question above, the student might solve for the diamond and choose 5.

Part 2: Number Puzzles Practice Test Script

The following script covers many issues that will help students do their best on the test. Read aloud the text printed in *blue italics*: these are directions to the students. Directions for you are in parentheses and should NOT be read aloud. Feel free to modify the script to ensure that students understand what they are supposed to do and how to do it.

It may be helpful to make copies of the practice questions in order to display them one at a time on an overhead projector. If this is not possible, hold up a copy of the student practice booklet and point to different parts of each practice question as you discuss them with the class.

(Make sure each student has a practice booklet. Then **SAY**:)

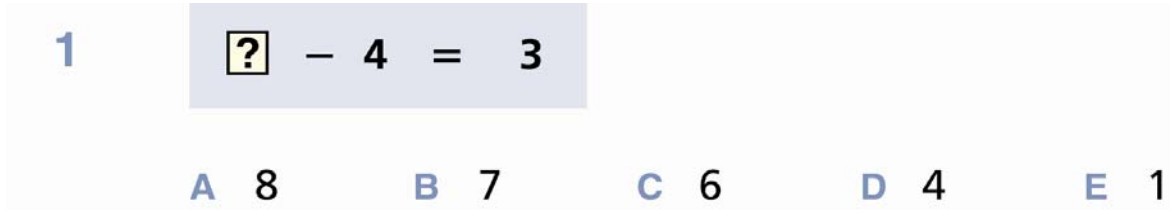
Open your practice booklet to page 2.

(Check that all students have the correct page.)

P1

Let's do the first practice question together.

(Hold up your copy of the practice booklet. Point to the numbers and signs in the shaded rectangle as you **SAY**:)



Which number goes in the square with the question mark?

(Encourage responses.)

Both sides of the equal sign must total the same amount. The right side is 3. The left side shows the square with the question mark minus 4. What number minus 4 will equal 3 so that the left side is the same as the right side?

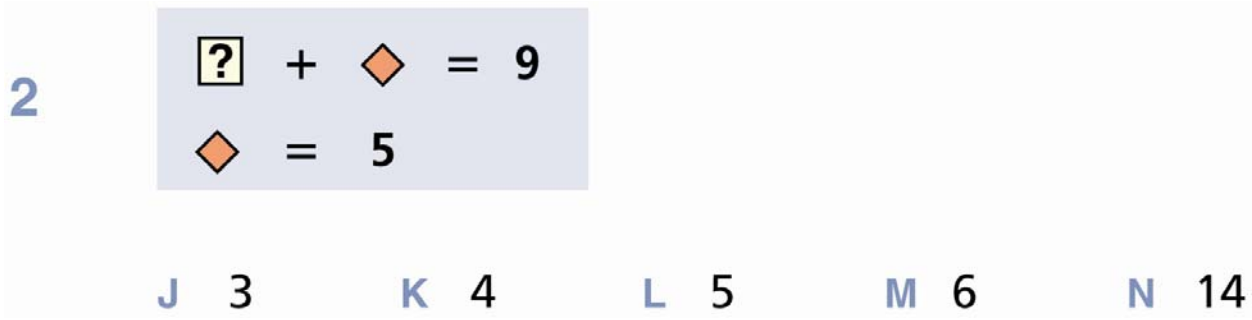
(Encourage responses. Then point to the answer choices as you **SAY**:)

The answer is 7 because 7 minus 4 is 3. So answer choice B is the correct answer.

(Check to make sure that all students have selected answer choice B.)

P2

Let's do the next practice question.



This question has two rows of numbers and shapes. In both rows, both sides of the equal sign must total the same amount.

Look at the top row of the question. The right side of the equal sign is 9. The left side shows the square with the question mark plus the diamond. So this side must total 9 to equal the right side of the equal sign.

But how much is the diamond?

(Pause.)

The second row shows that the diamond is equal to 5. If we replace the diamond in the first row with 5, then a number plus 5 must equal 9.

(Make sure that all students understand how to substitute the number 5 for the diamond. Provide another example if necessary.)

Which number goes in the square with the question mark?

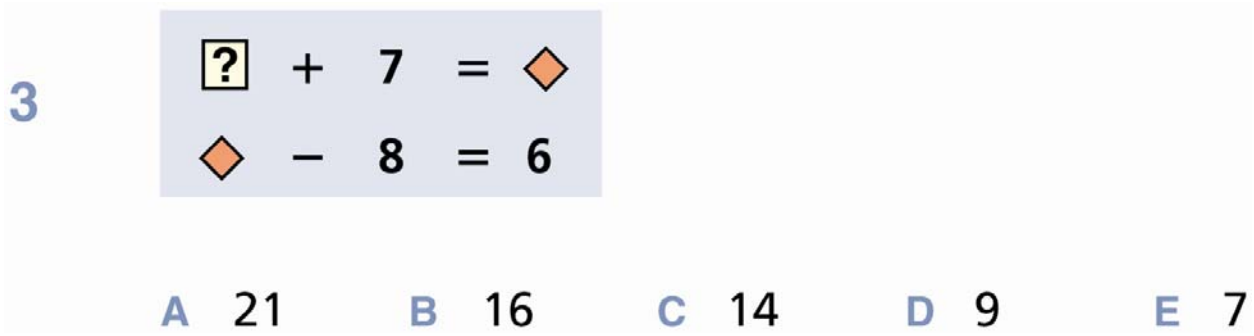
(Encourage responses. Then point to the answer choices as you **SAY**:)

The correct answer is 4 because 4 plus 5 is 9. Answer choice K is the correct answer.

(Check to make sure that all students have selected answer choice K. Encourage them to work from the bottom up rather than from the top down,)

P3

Let's do the next practice question.



Should we start with the top row or with the second row?

We should start with the second row because it will tell us how much the diamond is.

(Point to the second row as you **SAY**:)

The second row shows that the diamond minus 8 is equal to 6. So the diamond must be 14 since 14 minus 8 is 6.

If we replace the diamond in the first row with 14, then the square with the question mark plus 7 must equal 14.

Which number goes in the square with the question mark?

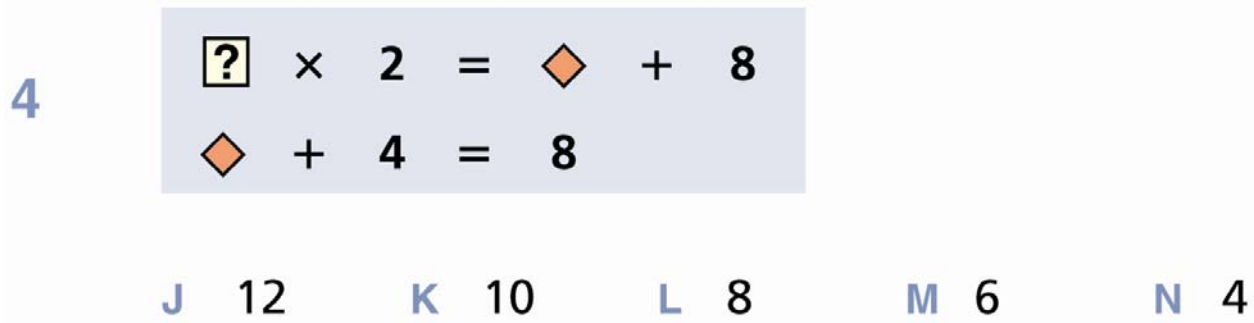
(Encourage responses. Then point to the answer choices as you **SAY**:)

The correct answer is 7 because 7 plus 7 is 14. Answer choice E is the correct answer.

(Check to make sure that all students have selected answer choice E.)

P4

Look at the next practice question. Try to solve this practice question on your own.



(Make sure students have enough time to solve the problem. Then **SAY**:)

Which number goes in the square with the question mark?

(Encourage responses.)

How did you solve this question?

(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

The second row shows that diamond plus 4 is equal to 8. So the diamond must be 4 since 4 plus 4 is 8.

If we replace the diamond in the first row with 4, then the square with the question mark multiplied by 2 must equal 4 plus 8, or 12.

Which number goes in the square with the question mark?

(Encourage responses. Then point to the answer choices as you **SAY**:)

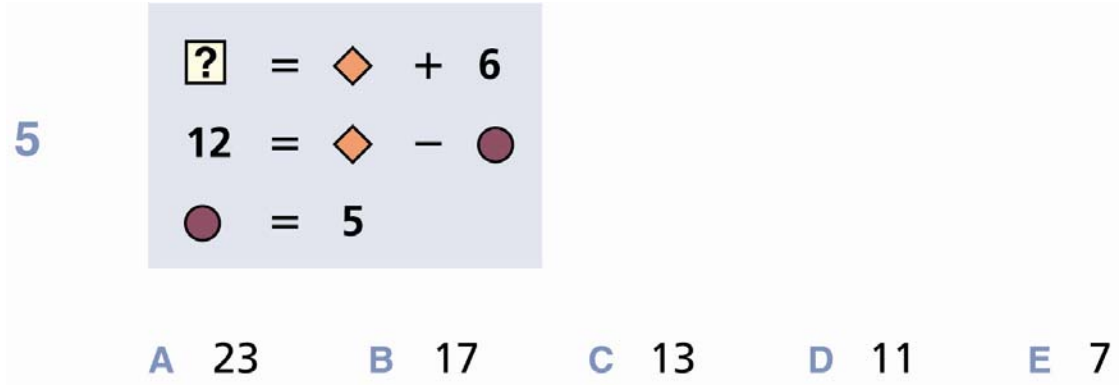
The correct answer is 6 since 6 multiplied by 2 is 12. So answer choice M is the correct answer.

(Check to make sure that all students have selected answer choice M.)

Turn to the next page.

P5

Let's do the next practice question.



There are three rows in this question. In the top row, the square with the question mark equals the diamond plus 6.

But how much is the diamond?

(Encourage responses. Then point to the second row as you **SAY**:)

In the second row, 12 equals the diamond minus the circle.

But how much is the circle?

(Encourage responses. Then point to the third row as you **SAY**:)

So we need to start with the bottom row to solve the problem.

The third row shows that the circle is the same as 5.

If we replace the circle in the second row with 5, then the diamond minus 5 must be equal to 12. So, the diamond must be 17 since 17 minus 5 is 12.

If we replace the diamond in the first row with 17, then the square with the question mark must be equal to 17 plus 6.

Which number goes in the square with the question mark?

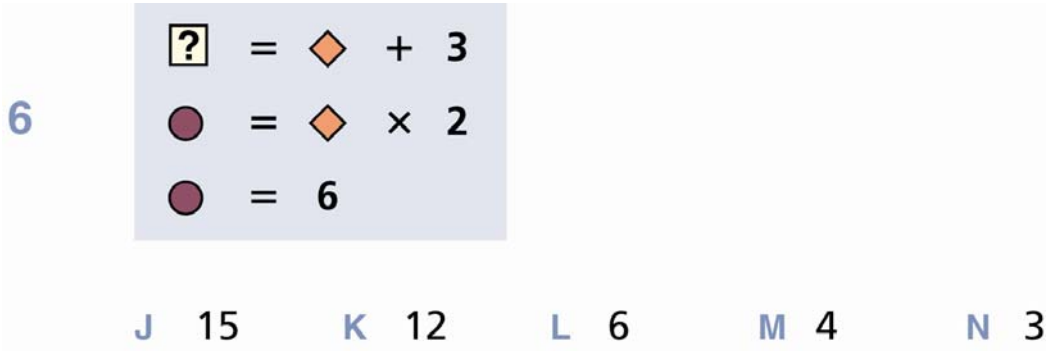
(Encourage responses. Then point to the answer choices as you **SAY**:)

The correct answer is 23, which is answer choice A, because 17 plus 6 is 23.

(Check to make sure that all students have selected answer choice A.)

P6

Look at the last practice question. Try to solve this practice question on your own.



(Make sure students have enough time to solve the problem. Then **SAY**:)

Which number goes in the square with the question mark?

(Encourage responses.)

How did you solve this question?

(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

The third row shows that the circle is the same as 6. If we replace the circle in the second row with 6, then the diamond multiplied by 2 must equal 6. So, the diamond must be 3 since 3 multiplied by 2 is 6.

If we replace the diamond in the top row with 3, then the square with the question mark must be equal to 3 plus 3.

Which number goes in the square with the question mark?

(Encourage responses. Then point to the answer choices as you **SAY**:)

The correct answer is 6, which is answer choice L, since 3 plus 3 is 6.

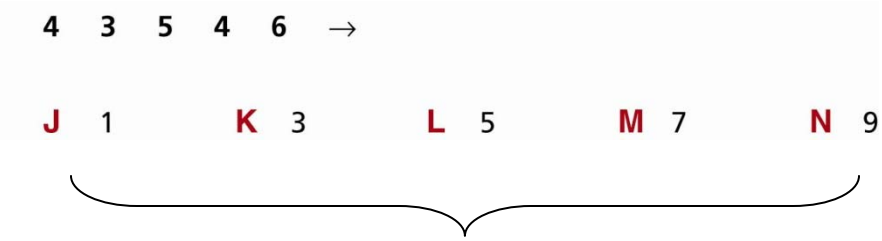
(Check to make sure that all students have selected answer choice L.)

Test 6: Number Series, Levels 10–12

Part 1: Overview of Number Series

The Number Series test requires the student to discover the pattern in a series of numbers and then infer the simplest logical extension of the pattern from the choices available. The questions in this subtest follow various kinds of patterns, so it is important to consider different patterns for each question. Students will be allowed to use scratch paper in the actual test.

In this test, students are asked to solve problems that look like this:



Which number should come next to continue the pattern?

When practicing the Number Series questions, encourage students to use these strategies.

- Look for repeating patterns in groups of adjacent numbers. If this is not successful, then look for patterns in every other number or every third number. For example, every other number in the series 4 3 5 4 6 increases by one.
- Apply the rule to determine which number will come next in the series.
- Eliminate answer choices that do not fit the rule.
- If more than one rule applies, use the simplest rule to decide what comes next (in most cases, alternate rules lead to the same answer).

Students at this level tend to make the following common mistakes.

- Students might use the wrong arithmetic operation. For example, the sample question above has a pattern that has two steps – first subtract1 then add 2. The student might confuse the two steps and add instead of subtract.
- Students may try to use the same pattern for every question and not attempt to find a new pattern for each question.

Part 2: Number Series Practice Test Script

The following script covers many issues that will help students do their best on the test. Read aloud the text printed in *blue italics*: these are directions to the students. Directions for you are in parentheses and should NOT be read aloud. Feel free to modify the script to ensure that students understand what they are supposed to do and how to do it.

It may be helpful to make copies of the practice questions in order to display them one at a time on an overhead projector. If this is not possible, hold up a copy of the student practice booklet and point to different parts of each practice question as you discuss them with the class.

(Make sure each student has a practice booklet. Then **SAY**:)

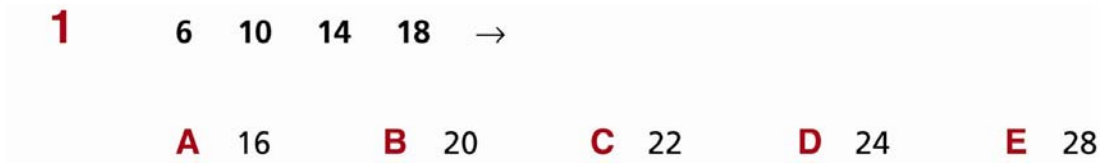
Open your test booklet to page 4.

(Check that all students have the correct page.)

In these questions, you must figure out what the pattern is for each series of numbers. Then choose the number that comes next in the pattern.

P1

Let's do the first practice question together.



(Point to the top row of numbers as you **SAY**:)

Look at the series of numbers in the top row. What pattern do the numbers in the series follow?

(Encourage responses.)

Each number is 4 more than the number before it. So the pattern is to add 4 to each number.

(Point to the answer choices as you **SAY**:)

Which number will come next in the series?

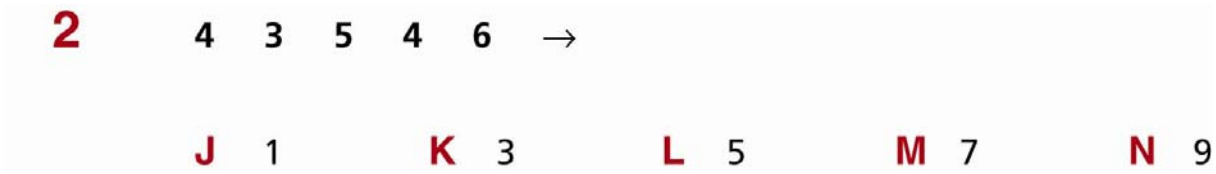
(Encourage responses.)

The pattern is to add 4 to each number. The last number is 18. 18 plus 4 is 22. So the next number in the series is 22 which is answer choice C.

(Check to make sure that all students have selected answer choice C.)

P2

Let's do the next practice question.



What pattern do the numbers in the series follow?

(Encourage responses.)

The first two numbers in the series are 4 and 3. To go from 4 to 3, we subtract 1. The next number is 5. To go from 3 to 5, we add 2. The next number is 4. To go from 5 to 4, we subtract 1. The next number is 6. To go from 4 to 6, we add 2. The pattern is: subtract 1, add 2, subtract 1, add 2, and so on.

(Point to the answer choices as you **SAY**:)

Which number will come next in the series?

(Encourage responses.)

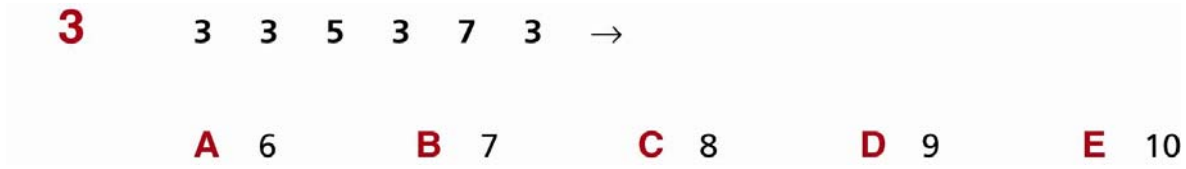
The next step is to subtract 1 from 6. 6 minus 1 is 5. So the next number in the series is 5 which is answer choice L.

(Check to make sure that all students have selected answer choice L.)

You can also solve this problem by looking for patterns in every other number. Every other number is increasing by 1. The answer should be 5 because 4 plus 1 is 5.

P3

Let's do the next practice question.



What pattern do the numbers in the series follow?

(Encourage responses.)

Look at every other number. Starting with the first number, what pattern does every other number follow?

(Encourage responses.)

After the first number, every other number increases by 2. Starting with the second number, what pattern does every other number follow?

(Encourage responses.)

After the second number, every other number simply repeats. So we can solve this problem by ignoring the repeating 3's.

(Point to the answer choices as you **SAY**:)

Which number will come next in the series?

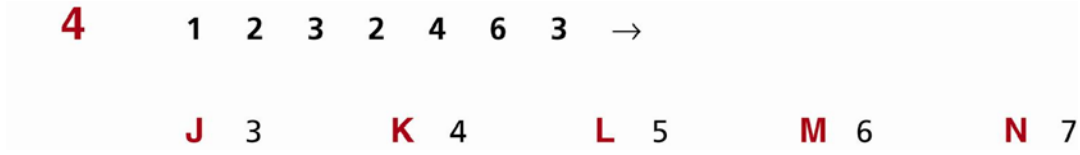
(Encourage responses.)

The next step is to add 2 to 7. So the next number in the series will be 9 because 7 plus 2 is 9. So answer choice D is correct.

(Check to make sure that all students have selected answer choice D.)

P4

Look at the next practice question. Try to solve this practice question on your own.



(Make sure students have enough time to solve the problem. Then **SAY**:)

Which number will come next in the series?

(Encourage responses.)

How did you solve this question?

(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

Look at the first three numbers: 1, 2, 3. Then look at the next three numbers: 2, 4, 6. How do the numbers in each set of three go together?

(Encourage responses.)

The second number in each set is the first number multiplied by 2. In the first set, the second number is 2, which is the same as 1 multiplied by 2. In the second set, the second number is 4, which is the same as 2 multiplied by 2.

The third number in each set is the first number multiplied by 3. In the first set, the third number is 3, which is the same as 1 multiplied by 3. In the second set, the third number is 6, which is the same as 2 multiplied by 3.

The last number in the series is 3. Which number must come next to continue the pattern?

(Encourage responses.)

Since 3 is the first number in a new set of three numbers, the next number must be 3 multiplied by 2. So the next number in the series is 6 which is answer choice M.

(Check to make sure that all students have selected answer choice M.)

P5

Now let's do the next practice question.

What pattern do the numbers in the series follow?

(Encourage responses.)

The numbers are increasing, but not by the same amount every time. To go from 2 to 7, we add 5. To go from 7 to 11, we add 4. To go from 11 to 14, we add 3. The amount that we increase the numbers by is decreasing by 1 every time. So the pattern is: add 5, add 4, add 3, and so on.

Which number will come next in the series?

(Encourage responses.)

The next step is to add 2 to 14. 14 plus 2 is 16. So the next number in the series is 16 which is answer choice B.

(Check to make sure that all students have selected answer choice B.)

P6

Look at the last practice question. Try to solve this practice question on your own.

(Make sure students have enough time to solve the problem. Then **SAY**:)

Which number will come next in the series?

(Encourage responses.)

How did you solve this question?

(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

Look at the top row of the question. What pattern do the numbers in the series follow?

(Encourage responses. Then point to the series and examine each successive pair of numbers. Write the operation that changes the first into the second in the space between them.)

In this series, sometimes the numbers increase and sometimes the numbers decrease.

The first two numbers are 13 and 6. To go from 13 to 6, we subtract 7.

To go from 6 to 12, we add 6. Then, we subtract 5. To go from 7 to 3, we subtract 4. Then, we add 3. To go from 6 to 4, we subtract 2.

So the pattern is: subtract 7, add 6, subtract 5, then subtract 4, add 3, subtract 2.

The operation that we use to change numbers follows this pattern: subtract, add, subtract, (Pause), subtract, add, subtract, and so on. So which operation do we use next?

(Encourage responses.)

To continue the pattern, we must subtract. But how much should we subtract?

(Encourage responses.)

The amount by which the numbers change also follows a pattern. That pattern is: 7, 6, 5, 4, 3, 2, and so on. So, these numbers are decreasing by 1 every time.

In this problem, we have to use both patterns to find the next number. Which number will come next in the series?

(Encourage responses.)

The next step is to subtract 1 from 4. 4 minus 1 is 3. So the next number is 3 which is answer choice L.

(Check to make sure that all students have selected answer choice L and that all students understand this reasoning.)