Opening up opportunities for gifted students to be creative in the mathematics classroom

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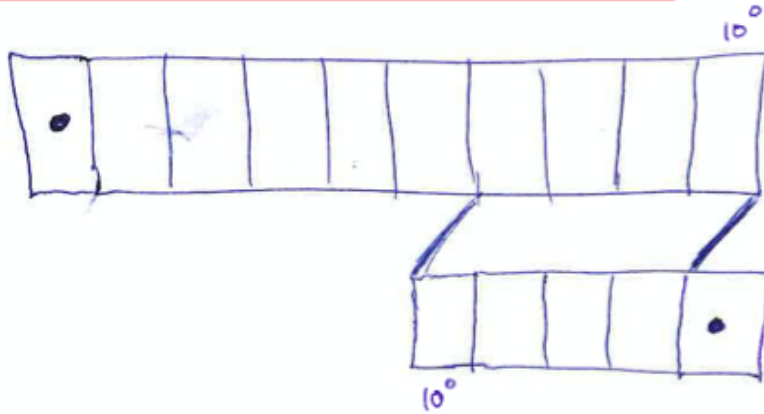
Middle School: Open Mathematics Inquiry (OMI)

- About a dozen 7th and 8th grade boys, 2 teachers

- Round table discussion with no wrong answers

- Incorrect thinking was amended by students with reference to other students' thinking and strategies

- Visual representations, stories, and calculations were developed and displayed for a gallery walk

- Lesson concluded with a multiple solutions task

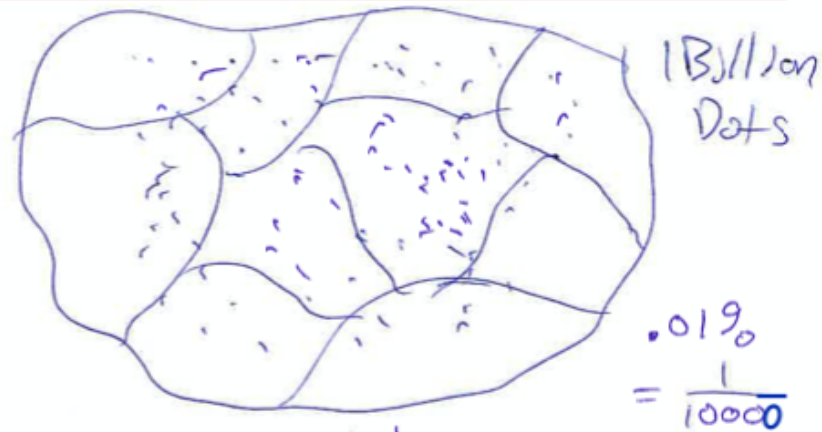# How would YOU calculate .01% of 1 Billion?

- Diamond Paper Activity

How Many different ways can WE calculate this value?

Give a visual Representation

10°

Give a different visual solution

1 Billion Dots

.01%

$= \frac{1}{10000}$

Subdivided into groups of 10000 yields some # of groups.
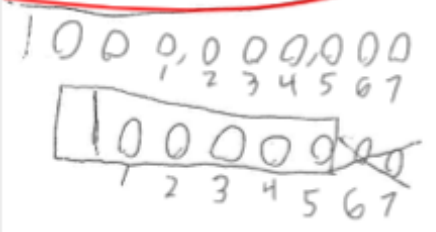
Calculate

.01% of

1 Billion

10°

Write a story

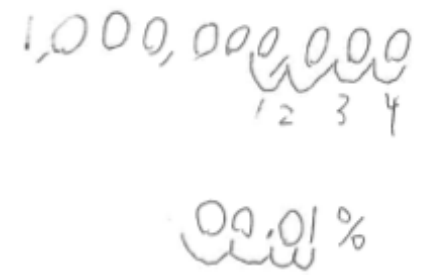A very very rich man has 1 Billion Foods and gives .01% of all his Foods to many individuals... 1 Food per individual.
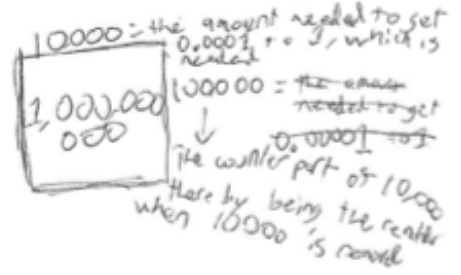
How many individuals does his .01% Feed?

Solve it numerically.

$$10^9 \cdot 10^{-4} = 10^5$$

100,000

Sample Results

1000,000,000
1 2 3 4 5 6 7

| 1 0 0 0 0 0 0 | |
1 2 3 4 5 6 7

1,000,000,000
1 2 3 4

00.01 %

So   0.01% of 1,000,000,000
= 100% of 100,000

Calculate
.01% of 1 billion

Amazon has 1 billion toys in their warehouse. 0.01% of them are going to a target store in arizona. How many toys will that target recieve?

0.01% of 1,000,000,000

```
          .0001
  ×
  1,000,000,000
  00 0 0 9 g 00 g 00 0
  00g 0 0 g 0 g g D g 0
+ 0900 000 00 0 0 00 0
  100 0 0 0.0 0 0 0
```

10000 = the amount needed to get 0.0001 to 1, which is needed

100 00 = the amount needed to get 0.00001 to 1

1,000,000 000

the smaller part of 10,000 there by being the remainder when 10,000 is removed
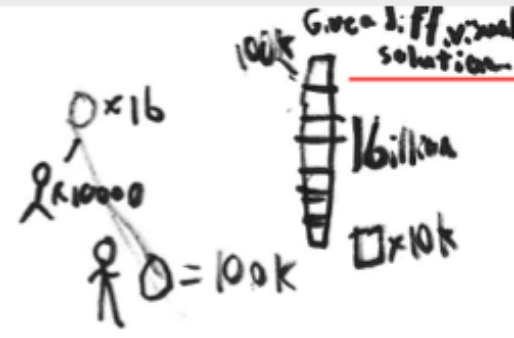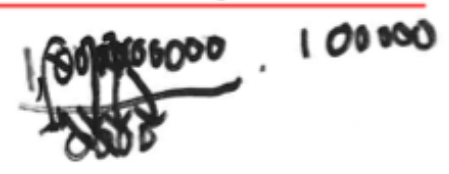
Calculate
0.1% of 1 billion

Jess has 2 billion grains of sand. Every day, 90% blow away. The first day, Jess awakens to 100 million grains. The next, he awakens to 10 million. The third, 1 million, and the fourth, 100 thousand. At this point, he has lost 99.99% of the grains, leaving 0.01% left.

×1000,000,000   ×100,000,000

♀ — 10% = ♀   — 10%

×10,000,000   ×100,000   ×1000
= ♀ — 10% = ♀ — 10% = ♀

10% = 0.01%

1% • 1000,000,000,
=
10,000,000

10,000,000 • 1/100 = 100,000

1,000,000,000 · 100,000

100k

□ × 16

& × 10000

⸜ O = 100k

⫿ 16 billion

□ × 10k

0.01% of 1 billion

NOT TO SCALE

$\frac{1}{100}$ (1%) of 1 billion

$\frac{1}{100}(1,000,000,000) =$ 10 million

$\frac{1}{100}(10,000,000) = 100,000$

& billion

$\frac{1}{100}$ of 10 mill, 0.01% of 1 bil

10 million
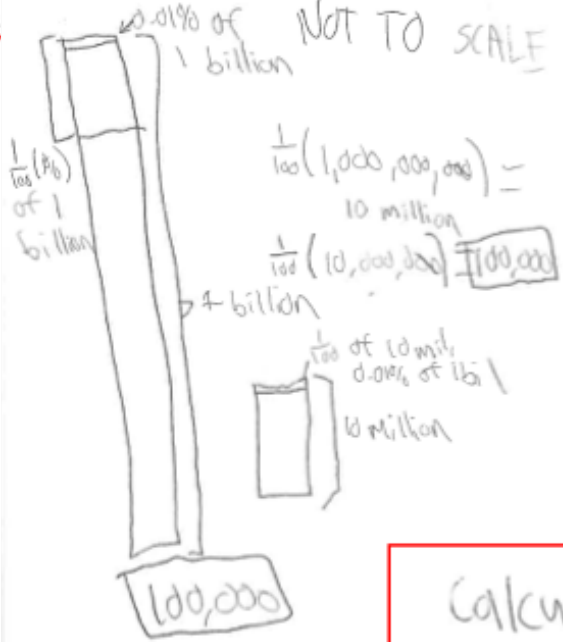
100,000

$\left(\frac{1}{100}\right)\left(\frac{1}{100}\right)\left(\sqrt{1 bil}\right)\left(\sqrt{1 bil}\right) =$

$\frac{1}{10000}(1,000,000,000) =$

100,000

Calculate .01% of 1 billion.

Calculate .01% of 1 Billion

The host of a famous TV show has 1 billion dollars to split among 10,000 contestants equally. Each contestant gets 100,000 dollars.

$1,000,000,000 \cdot \frac{.01}{100}$

$= 10^9 \cdot 10^{-4}$

$= 10^5$

$= 100,000$

$\frac{1}{100} \cdot \frac{1}{100} = \frac{1}{10,000}$

$\frac{1}{10,000}(1,000,000,000) =$

100,000

Once upon a time there was a billion rubik's cubes. Only 0.01% of them were solved. A man named Sal v the math tried to figure it out. He found 1% of a billion, which was ten million. 0.01 is 1% of 1, so he found 1% of ten million, which was 100,000, the answer.

Why Use Open Inquiry?

Yeo, J. B. W. (2015). Development of a Framework to Characterise the Openness of Mathematical Tasks. *International Journal of Science and Mathematics Education*, *15*(1), 175–191. https://doi.org/10.1007/s10763-015-9675-9

- Closed vs. Open: *Answer, Goal, Method, Extension* (Yeo 2015)
- Integrated lesson across disciplines
- Opportunity for students to be artistic/verbal
- Ill-structured tasks are ***fun*** and ***creative*** for ***gifted*** students
- Promotes student development in *flexibility, fluency*, and *novelty*

Teaching Constructions and Loci: WHEW!

- High School Geometry utilizing a *Flipped Classroom* model
- Introductory *videos* on perpendicular and angle bisectors
- Students self reported by **color** at beginning of class
- Different entry points based on real-time data
- Low-Floor/High-Ceiling task
- **W**HAT: Guided notes on definitions
- **H**OW: Procedural skill of constructing bisectors
- **E**XPLORE: Investigative task of noticing and wondering
- **W**HY: Problem posing extensions, mathematical proof

After watching the videos on Constructions and Loci, please select the answer below that best describes your current understanding.

○ I haven't finished watching videos 2.3 and 2.4 yet.
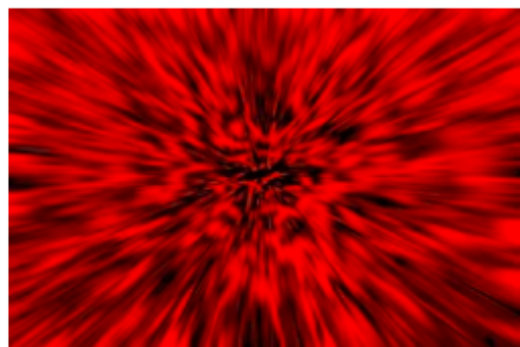
○ I don't understand WHAT bisectors are yet.

○ I know that angle and perpendicular bisectors are loci, but don't know HOW to construct them yet.

○ I can construct both types of bisectors, I'm ready to EXPLORE more interesting questions.

Constructions and Loci Check-IN

On Angle Bisectors and Perpendicular Bisectors

22 responses

Accepting responses ⬤

Summary | **Question** | **Individual**
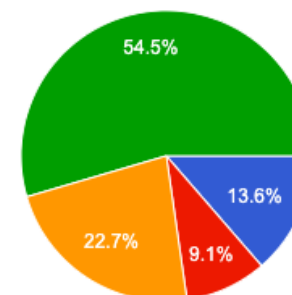
After watching the videos on Constructions and Loci, please select the answer below that best describes your current understanding.

22 responses

- 🔵 I haven't finished watching videos 2.3 and 2.4 yet.
- 🔴 I don't understand WHAT bisectors are yet.
- 🟠 I know that angle and perpendicular bisectors are loci, but don't know HOW to construct them yet.
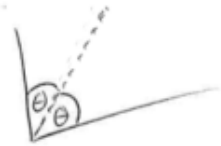- 🟢 I can construct both types of bisectors, I'm ready to EXPLORE more interesting questions.

54.5% · 13.6% · 9.1% · 22.7%

Guided Notes

Procedural Skill

Investigative Task

EXPLORE

1. Draw a triangle below the line

2. Bisect all of the angles

What do you notice?
what do you wonder?

3. Bisect all of the sides
   perpendicularly

What do you notice?
what do you wonder?

pro tip: do 1., 2., and 3. in different colors

Why?

why did this happen?
will it always happen? Prove it!
Does the type of triangle matter?
(Acute, obtuse, right ... scalene, isosceles, equilateral)
why or why not?

Does a proof exist without cases?

Extensions
Problem Posing

1. Draw a triangle to the right

2. Bisect ALL of the angles

3. ⊥ Bisect ALL of the sides

4. What do you notice?

5. What do you wonder?

I notice the intersection
of all angle bisectors is a
single point. And the
intersection of the 3 ⊥ bisectors
is a different point.

I wonder if this will always
happen.
I wonder why this happened.

Benefits	Challenges
Allows ALL students to begin where they are comfortable with new mathematics content	Diversity of learners background knowledge/ability Accelerated curriculum How to provide both enrichment and remediation
Peer collaboration during HOW and EXPLORE phases for GREEN/YELLOW reporters Teacher led instruction and support for RED reporters	Some students may arrive to class unprepared to participate in the days activity (PURPLE/BLUE)
Investigative exploration tasks using an open middle allows students time for discovery and deep conceptual understanding	Ensuring students are thoroughly engaged Looking for all special cases (e.g. scalene, right, acute, obtuse triangles)
Problem posing tasks provide high ceiling opportunities for extension of mathematics curriculum in the diverse classroom	Encouraging thinking when students are stuck without providing answers for them
Novelty and originality are both called for in the solutions to proof based questions in the WHY phase of the task	This can be extremely difficult for students being introduced to mathematical proof for the first time (High Complexity)

Other Open Inquiry Strategies...

- Example Generation
  - Directly asking students to "create" something to satisfy a mathematical existence statement

- Question Writing
  - Have students write their own real-world modeling problems
    - Example: Related Rates in Calculus

If the Antares star with a radius of $473 \cdot 10^6$ km went supernova, at what rate would the volume of the gas cloud produced by the explosion expand if material is ejected at a rate of 25,600 km/sec?

$r \cdot 473 \cdot 10^6$ km

Antares is the 15th brightest star in the sky. It is located in the Scorpius constellation.

Each year, the moon moves approximately 3.8 cm away from the Earth. Assume that at t(0) the moon's orbit around the Earth is a perfect circle with a radius of 3.48 x 10^8 m. What is the rate of change in the circumference of the moon's orbit when 3 years have passed?

Questions, Comments, Further Discussion

- <u>Other Resources</u>
  - [Which One Doesn't Belong](#)
  - [Number](#)/[Dot Talks](#)
  - [Open Middle](#)/[3-Act Tasks](#)
  - [Model Eliciting Activities](#)/[Case Studies for Kids](#)

Let me know your thoughts about these lesson frameworks:

I encourage you to build your own OMI and WHEW to use in the classroom and look forward to hearing about how it goes

- Email Casey Warmbrand: cwarmbra@asu.edu